IDENTIFICATION D’UN ÉTALEMENT DIRECTIONNEL CONSERVATIF POUR REALISER DES POLAIRES DE ROULIS EN ÉTAT DE MER REEL

IDENTIFICATION OF A CONSERVATIVE SPREADING ANGLE TO REALIZE OPERATIONAL ROLL POLAR PLOTS CONSIDERING REAL SEA STATES

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Résumé

L’étalement directionnel caractérisant un état de mer est une entrée classique des simulateurs hydrodynamiques temporels. Cet étalement est difficilement estimé depuis la passerelle des navires. Toutefois, celui-ci a une influence directe sur les mouvements du navire. Il est donc nécessaire de le renseigner avec la plus grande pertinence possible lorsque des simulations sont réalisées afin d’évaluer de la vulnérabilité du navire. Lors de la réalisation de polaires de roulis, une unique valeur de l’étalement directionnel est considérée afin de limiter les temps de calculs. Cette étude a pour objectif d’identifier la valeur de l’étalement directionnel le plus conservatif à considérer lors de la réalisation de ces polaires. Un état de mer modélisé avec un spectre de Pierson-Moskowitz est considéré comme référence. Des états de mer étalés équivalents sont construits. La hauteur significative de chaque spectre est calculée en utilisant une fonction d’étalement en cos^8 tel que recommandé dans l’évaluation du risque de roulis paramétrique par le Bureau Veritas (NR 667, [1]). Chaque état de mer étalé équivalent est validé par comparaison de
l’énergie cumulé avec celle de l’état de mer de référence. L’étude est réalisée à partir des résultats de simulations du navire porte conteneur de classe C11 conduites en 6 degrés de libertés sur l’état de mer monodirectionnel de référence et les états de mer étalés équivalents. La comparaison des amplitudes de roulis obtenues permet d’identifier l’étalement directionnel le plus conservatif. Les phénomènes rares tel que le roulis paramétrique ou le roulis synchrone sont traités avec une attention particulière.

**Summary**

The sea state spreading angle is a common input value in time domain simulations. It is hardly operationally evaluated from the bridge. However, it has a direct influence on the ship motion. Therefore, it is necessary to specify it as relevantly as possible when conducting simulations to evaluate the vulnerability of a vessel. When building operational roll polar plots, a unique value of the spreading angle is used to limit computational time. This study aims to identify the value of the most conservative spreading angle when building roll polar plots. A sea state modelized with a Pierson-Moskowitz spectrum is considered as reference. An equivalent spread sea state, constituted of several monodirectional sea states from different directions, describing altogether the same energy, is built. The significant height of each spectrum is calculated considering a cos^8 spreading function such as recommended in the parametric roll assessment NR 667 [1]. Each resulting equivalent spread sea state is validated by comparison of its cumulated energy with the one of the reference sea state. The study is conducted by 6-degree-of-freedom simulations on the C11-class container ship on both the reference monodirectional sea state and the equivalent spread sea states. The comparison of the resulting roll amplitudes leads to identify the most conservative spreading angle. Rare phenomenon such as parametric or synchronous roll are treated with special care.
I - Introduction

The analytical definition of a sea state is quite complex to reflect its encountered diversity. Thus, sea states are defined by their spectrum, significant wave height and period on which a spreading function is optionally added. The spreading function reflects how the sea state is spread on both sides of the main wave direction. It is associated with the spreading angle which is the angle on which this spreading occurs on either side of the main wave direction. Operationally, several wave systems may appear such as sea and swell coming from different directions. Each wave system is described by a sea spectrum, a main direction, and a spreading angle. The sea spectrum is too complex to be identified by the officer of the watch. However, he/she can estimate the wave period and the wave height. Furthermore, the officer of the watch cannot identify the spreading angle from the bridge; only the main direction of the wave is estimated. In these conditions, the information provided to the officers of the watch on the possible vessel roll motions based on his/her evaluation of the sea state are to be the most conservative. Therefore, when evaluating the vessel seaworthiness [7] by realizing operational roll polar plots, simulations in 6 degrees of freedom (DoF) should be conducted considering a relevant value of the spreading angle.

The aim of this paper is to define the value of the most conservative spreading angle. This study, considering fully developed sea states, comes in addition to a preliminary study considering a sinusoidal wave as reference [8].

When conducting time domain simulations, the spreading is defined as the spreading angle (denoted by $\Delta \alpha$), the discrete number of considered waves directions and the associated spreading function. A “$\cos^n$” spreading function is commonly considered, with $n = 8$ such as proposed by Bureau Veritas [1]. An increase of the number of wave directions leads to an increase of the computation time of the simulation.

First the method to generate equivalent spread sea state developing the same energy to the vessel is proposed. Then, the influence of the spreading angle on the vessel roll motion is presented based on 6-degree-of-freedom simulations realized with the time domain solver Fredyn [3]. The results are compared and discussed, and the most conservative spreading angle is identified.

II – Reference and equivalent sea states

II – 1 Reference sea state

A monodirectional sea state modeled with a Pierson Moskowitz spectrum (Equation (1), [9]) is considered as reference. The sea state is defined with a reference significant height (denoted by $H_S$) and a reference up-crossing period (denoted by $T_Z$).

$$ S(\omega) = \frac{1}{4\pi} H_S^2 \left(\frac{2\pi}{T_Z}\right)^4 \omega^{-5} e^{-\frac{1}{2\pi T_Z^4}} \omega^{-4} $$

(1)

Where, $\omega$ denotes the frequency.

Since the spectrum provides the distribution of wave energy as a function of the wave frequency, the area under the spectrum reflects the total energy developed by the sea state (denoted $E_0$ for the reference sea state). In Equation (1), it is observed that the spectrum amplitude is proportional to the square of the significant wave height for a given up-crossing period. As well, the area under the spectrum is proportional to the square of significant wave height for a given up-crossing period. Thus, the energy developed by the spectrum is proportional to the square of significant wave height for a given up-crossing period.

II – 2 Equivalent sea state
The energy developed by the reference sea state is distributed to each component of the spread sea state. The number of directions (denoted by N) is calculated depending on the spreading angle (Δα) to obtain a maximum spacing of 10 degrees between two adjacent directions. N shall be odd to keep a wave component in the main direction. The resulting spacing (denoted by δα) is calculated using Equation (2). As example, for a spreading angle of ±30 degrees, N is equal to 7 and δα is equal to 10 degrees. The main direction is identical to the one of the reference sea state and the other directions are calculated relative to this main direction, using the spreading angle and the number of considered directions.

\[
\delta \alpha = \frac{2 \Delta \alpha}{N - 1} \tag{2}
\]

The energy of the reference sea state E₀ (image of the area under the spectrum) is distributed in the N directions based on a cos^n spreading function [2]. The energy of reference is the one developed by the entire sea spectrum. N areas are defined within the range [-π/2; +π/2] under the cos^n function. The sum of the N areas under the spreading function is equivalent to the total energy E₀. Each area is associated to its own direction (denoted by \(\alpha_i\) in radian, where i defines the direction index) and to its percentage \(\chi_i\) of the total area (Equation (3)).

\[
\chi_i = \frac{A_i}{A_{tot}} \tag{3}
\]

Where \(A_i\) denotes the area under the cos^n spreading function associated to the \(i^{th}\) direction, \(A_{tot}\) denotes the overall area under the cos^n spreading function from - \(\pi/2\) to \(\pi/2\).

Figure 1 provides a graphic representation of the areas to consider associated to the wave directions for a spreading angle of ±30 degrees and 7 wave directions (N = 7) with a cos^8 spreading function. In this example, the main wave direction is equal to 0.

A sea state of period equal to the one of the reference \(T_Z\) and of significant height \(H_{Si}\) is associated to each direction \(\alpha_i\). The spectrum of the sea state associated to the \(i^{th}\) direction is defined using Equation (1). \(\chi_i\) is calculated for each direction as presented above, and the corresponding significant wave height \(H_{Si}\) is calculated considering this energy repartition. Since the energy of the spectrum is proportional to the square root of the height of the sea state, Equation (4) permits to calculate the significant wave height associated to each direction based on the both the significant height of the reference sea state (denoted by \(H_S\)) and of the energy repartition.
\[ H_{Si} = \sqrt{x_i} H_S \] (4)

The monodirectional sea spectrum for each direction is defined using Equation (1). The sum of the areas under the N spectra, defining all together the spread sea state, is equal to the area under the reference spectrum, defining the same energy on a considered surface.

An example is provided for a reference sea state of significant height equal to 5 m and of up-crossing period equal to 12 s (the mean period is equal to 13.03 s). The reference sea spectrum is built using Equation (1) and the area under this spectrum is calculated (Figure 2). The area under this spectrum between 0.24 and 0.85 rad.s\(^{-1}\) is equal to 1.491 m\(^2\) (red area). The spreading angle is set to \(\pm 30\) degrees. 7 waves directions are considered with a \(\cos^8\) function. The 7 sea state directions and the energy repartition for each direction are calculated using the method presented above. The significant height of each sea state in each direction is calculated using Equation (4). Each resulting sea spectrum is defined using Equation (1) and the area under each spectrum is calculated (Table 1). The sum of the areas under the spectra associated to all directions is equal to 1.491 m\(^2\). Thus, both the reference and the equivalent spread sea state develop the same energy. Finally, identical calculation is performed with the data extracted from the output file of the time-domain solver Fredyn. The results lead to the same energy. This validates the implementation of the spread sea state in the time-domain solver.

![Figure 2. Pierson-Moskowitz sea spectrum, T\(_Z\) = 12 s, H\(_S\) = 5 m](image)

<table>
<thead>
<tr>
<th>Heading [deg]</th>
<th>(H_{Si} [m])</th>
<th>Area under (i^{th}) spectrum [m(^2)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>-30</td>
<td>0.043522</td>
<td>0.000113</td>
</tr>
<tr>
<td>-20</td>
<td>0.708589</td>
<td>0.029958</td>
</tr>
<tr>
<td>-10</td>
<td>2.424645</td>
<td>0.350772</td>
</tr>
<tr>
<td>Main direction</td>
<td>3.497744</td>
<td>0.729969</td>
</tr>
<tr>
<td>+10</td>
<td>2.424645</td>
<td>0.350772</td>
</tr>
<tr>
<td>+20</td>
<td>0.708589</td>
<td>0.029958</td>
</tr>
<tr>
<td>+30</td>
<td>0.043522</td>
<td>0.000113</td>
</tr>
<tr>
<td>sum</td>
<td></td>
<td>1.491655</td>
</tr>
</tbody>
</table>

Table 1. Example of equivalent spread sea state
III - Influence of the spreading angle on the roll motion

III – 1 Simulation conditions

Simulations are conducted on the container ship for several spread sea states (Table 2), using the time-domain solver Fredyn [3]. The vessel selected for this study is the C11-class container ship of length equal to 262 m, known for her vulnerability to parametric roll [4]. Each simulation is one-hour long. A unique simulation is not sufficient to obtain a representative maximum roll angle for each spread sea state, loading condition, vessel heading and speed. Thus, the median of the maximum roll angle observed on 20 simulations of 1 hour with different seeds is considered as recommended by Bureau Veritas [1].

<table>
<thead>
<tr>
<th>Case n°</th>
<th>Spreading angle [deg]</th>
<th>Number of waves</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>Reference</td>
</tr>
<tr>
<td>2</td>
<td>± 30</td>
<td>7</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>± 90</td>
<td>21</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 2. Set of waves parameters

In this study, the draught is set to 12 metres. Two loading conditions representing a KG of 17 and 18 metres are assessed. Two reference sea states are considered and described in Table 3. The probability of occurrence is the one provided in the IACS Rec.34 [5].

<table>
<thead>
<tr>
<th>Sea state n°</th>
<th>Hₜ [m]</th>
<th>Tₚ [s]</th>
<th>Occurrence Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>13</td>
<td>300 / 100 000</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>10</td>
<td>468.9 / 100 000</td>
</tr>
</tbody>
</table>

Table 3. Sea state definition

III – 2 Roll polar plots

Roll polar plots, representing the median 1-hour maximum roll angle of 20 simulations, are realised for the spreading angles considered in Table 2. The speed discretisation is 2 m.s⁻¹ from 2 to 10 m.s⁻¹ and the heading discretisation is 15 degrees from head sea to following sea. Half of the roll polar plot is calculated since the results are symmetrical. This represents 1300 hours of simulations for each combination of loading and environmental conditions. The method used to build the equivalent spread sea states developing the same energy is validated for each case in the time-domain solver by comparison of the total energy with the one of reference. The median 1-hour maximum roll angle for each combination of course and speed is the one displayed on the roll polar plots.

The roll polar plots obtained on the sea state number 2 (Table 3) with a KG of 18 metres are presented hereafter. Figure 3 presents the roll polar plot obtained without spreading, Figure 4 presents the roll polar plot obtained for a spreading of ± 30 degrees and Figure 5 presents the roll polar plot obtained for a spreading of ± 90 degrees. The black line in Figure 3 (without spreading) corresponds to the combinations of course and speed associated to the highest risk of encountering parametric roll (the natural roll period is twice the encounter period), based on the period of peak of the monodirectional spectrum and the formula of the encounter period provided by the International Maritime Organization [6].
Figure 3. Roll polar plot, case n°1 (reference spectrum)

Figure 4. Roll polar plot, case n°2 (spreading ± 30 degrees)

Figure 5. Roll polar plot, case n°3 (spreading ± 90 degrees)
Figure 3 to Figure 5 shows that the increase of the spreading angle tends to increase the width of combinations of course and speed leading to heavy roll motions. However, this study shows that the maximum roll angles, reached for combinations of course and speed where parametric or synchronous roll appear, decrease with the increase of the spreading angle. This conclusion was expected by the authors since the energy exciting the ship in parametric and synchronous mode is spread in several directions. This conclusion is contrary to the one observed in sinusoidal waves [8].

The median 1-hour maximum roll angles calculated for each combination of course and speed are compared for each spreading angle. The results are presented in Table 4. The column “±90° v. ±30°” presents the percentage of simulations for which the maximum roll angle observed with a spreading angle of ±90 degrees is larger than the one obtained with a spreading angle of ±30 degrees. As well, columns “±90° v. 0°” and “±30° v. 0°” present the percentage of simulations for which the maximum roll angle observed with a spreading angle of ±90 and ±30 degrees is larger than the one obtained without spreading angle, respectively. It is observed that both ±90 and ±30 degrees spreading angles lead in average to conservative roll amplitudes compared to the one without spreading (73.9 % of the cases). In addition, the spreading angle of ±90 degrees leads in average to results more conservative than the spreading angle of ±30 degrees (66.7 % of the cases).

This study leads to the conclusion that the most conservative spreading angle is ±90 degrees in most of the cases. Thus, a spreading angle of ±90 degrees should be considered as the most conservative one when realising simulations to build operational roll polar plots.

<table>
<thead>
<tr>
<th>KG [m]</th>
<th>Sea state case</th>
<th>±90° v. ±30°</th>
<th>±90° v. 0°</th>
<th>±30° v. 0°</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>1</td>
<td>69.2%</td>
<td>70.8%</td>
<td>76.9%</td>
</tr>
<tr>
<td>17</td>
<td>2</td>
<td>66.2%</td>
<td>72.3%</td>
<td>67.7%</td>
</tr>
<tr>
<td>18</td>
<td>1</td>
<td>67.7%</td>
<td>78.5%</td>
<td>70.8%</td>
</tr>
<tr>
<td>18</td>
<td>2</td>
<td>66.2%</td>
<td>73.8%</td>
<td>80.0%</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td>66.7%</td>
<td>73.9%</td>
<td>73.9%</td>
</tr>
</tbody>
</table>

Table 4. Compared percentage of mean maximum roll angle

**IV – Conclusion**

The aim of this paper is to identify the most conservative spreading angle to be considered when realising operational roll polar plots. This conservative spreading angle is obtained by comparison of the influence of equivalent spread sea states on the ship motion. Equivalent spread sea states are built to develop a total energy equal to the one of the reference sea state. Then, roll polar plots for the C11-class container vessel are realised, considering the median 1-hour maximum roll angle observed from twenty 6-degree-of-freedom simulations for each combination of course and speed. Roll polar plots are generated for each spreading angle and compared with each other. The roll polar plots show that the maximum roll angle due to parametric roll reduces as the spreading angle increases. The roll angle reached on a set of waves built for a spreading angle of ±90 degrees is larger in 73.9 % of the cases than the one reached when no spreading is considered, and larger in 66.7 % than the one reached when a spreading angle of ±30 degrees is considered. This study validates the use of a conservative spreading angle of ±90 degrees for the C11-container vessel. Further studies can be conducted on other hull shapes or types of ships to state is this conclusion can be generalized.
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References